Summer Work

SUPA Calculus 3

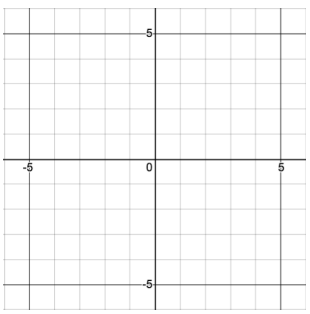
Please complete all problems and be ready to turn it in at the beginning of the school year. You can use any resources online for reference, graphing, or calculations, but please show any necessary work. A useful graphing tool is `desmos.com/calculator`.

You can email Mr. Caulfield at `mcaulfield@rih.org` with any questions.

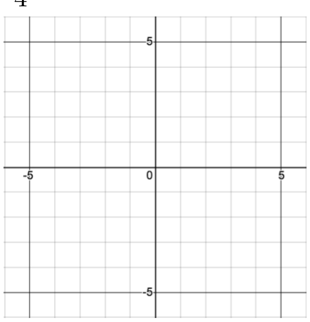
Conic sections

For each of the following, graph the conic section and identify the type.

1. $\dfrac{x^2}{9} + \dfrac{y^2}{16} = 1$

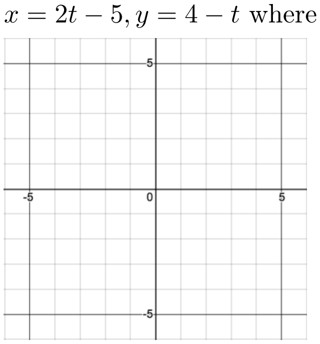


2. $\dfrac{x^2}{4} - y^2 = 1$



Parametric equations

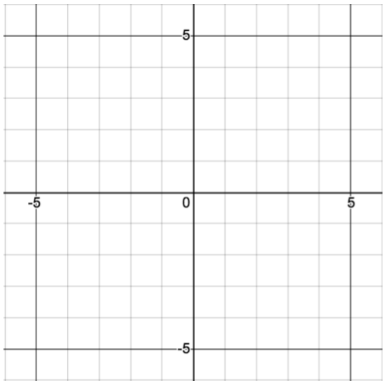
3. Sketch the curve. Indicate the direction of increasing t with an arrow.
   $x = 2t - 5, y = 4 - t$ where $0 \leq t \leq 4$.



4. For the curve:

   $x = 5\cos t, y = 3\sin t$ where $0 \leq t \leq 2\pi$.

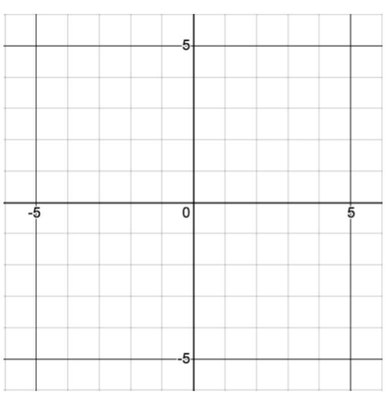
   Sketch the curve. Indicate the direction of increasing t with an arrow. Then find, in point-slope form, the equation for the tangent line to the curve at $t = \pi/6$, and graph the tangent line.



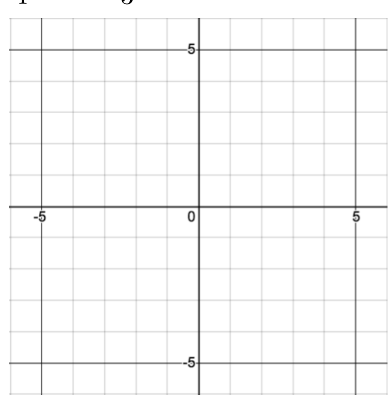
Polar coordinates

Sketch each inequality as a region in the plane.

5. $1 \leq r \leq 3$



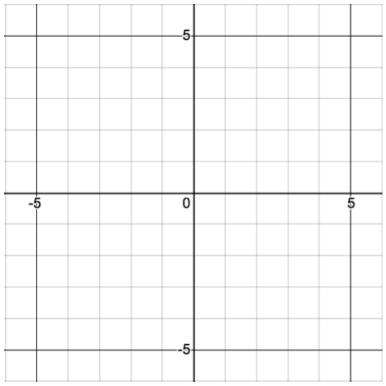
6. $\frac{\pi}{4} \leq \theta \leq \frac{\pi}{3}$



7. Sketch the curve:

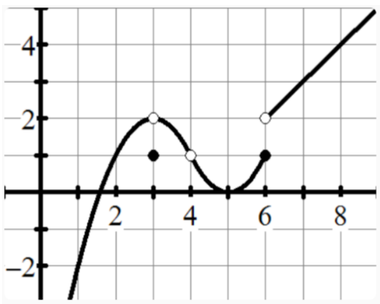
$r = 4\sin\theta$

$0 \le \theta \le \pi$



Limits

8. Consider the graph of the function $f(x)$ below:



Find each of the following, or determine if it's undefined.

(a) $f(3)$

(b) $f(4)$

(c) $f(6)$

(d) $\lim_{x\to 3} f(x)$

(e) $\lim_{x\to 4} f(x)$

(f) $\lim_{x\to 6} f(x)$

(g) $\lim_{x\to 6^+} f(x)$

(h) $\lim_{x\to 6^-} f(x)$

9. Evaluate the limit **without using L'Hopital's Rule**:

$$\lim_{x\to 0} \frac{(x-1)^2 - 1}{3x}$$

Derivatives

10. Find the derivative of the function $f(x) = e^{2x}x^{10}$

11. Find the derivative of the function, $f(x) = \dfrac{x^2 + 5x}{2x + 1}$

12. Find, in slope-intercept form, the equation for the tangent line to $f(x) = x^3 + 2x^2 - 4$ at $x = -1$.

13. Find the derivative of the function, $f(x) = \sin(\ln(2x))$.

14. Find and classify all critical points of the function:
$f(x) = x^3 - 12x + 1$

Integrals

Evaluate each of the following:

15. $\displaystyle\int_0^1 x^2(1+2x^3)^5dx$

    Hint, use U-substitution, you do *not* have to multiply out $(1+2x^3)^5$.

16. Find the arc length of the curve $y=\dfrac{2}{3}(x-1)^{\frac{3}{2}}$ on the interval: $0\le x\le 9$.

17. Consider the integral $\int_0^7 f(x)dx$, where:

$$f(x) = \begin{cases} 3x & 0 \leq x \leq 2 \\ 6 & 2 \leq x \leq 7 \end{cases}$$

Sketch the function on the interval $0 \leq x \leq 7$ and shade an area representing the intergral. Then use geometry **not calculus** to evaluate the integral.

18. Evaluate $\int_1^2 x^4 \log_2(x)dx$

Hint: Use change of base and integration by parts.

19. The velocity of a particle is described by the function, $v(t) = 6t + \sin(t)$.

(a) If the initial position of the particle is $x(0) = 10$, find $x(t)$, the position of the particle as a function of time.

(b) Find the acceleration of the particle at $t = \pi$.

20. Find the area enclosed by the two curves, $y = x^2 - 4x + 6$ and $y = x + 2$.